Name of the Student:_____
Max. Marks : 24 Marks **Time : 24 Minutes**

Q1.

The Global Positioning System (GPS) uses satellites to support navigation on Earth.

(a) One GPS satellite is in a circular orbit at a height h above the surface of the Earth. The Earth has mass M and radius R.

Show that the angular speed of the satellite is given by

$$\omega = \sqrt{\frac{GM}{(R+h)^3}}$$

(2)

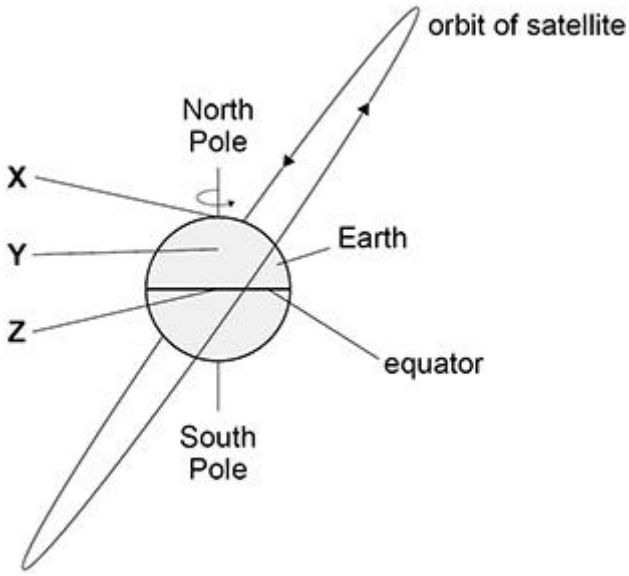
(b) Calculate the orbital period of the satellite when h equals 2.02×10^7 m.

orbital period = _____ s

(2)

(c) The figure below shows the orbital plane of the satellite inclined at an angle to the equator. **X**, **Y** and **Z** are locations on the Earth.

X is at the North Pole, **Y** is on a high mountain and **Z** is on the equator.



The satellite is to be launched from one of the locations.

State and explain which launch site **X**, **Y** or **Z** minimises the amount of fuel required to send the satellite into its orbit.

_____

_____

_____

_____

_____

(2)

(d) The satellite has a mass of 1630 kg.

Calculate the gravitational potential energy of the satellite when in the orbit in part (b).

gravitational potential energy = _____ J

(2)

(e) A different satellite is in a higher circular orbit.

Explain how the linear speed of this satellite compares with the linear speed of the satellite in part (a).

_____

_____

_____

_____

_____

_____

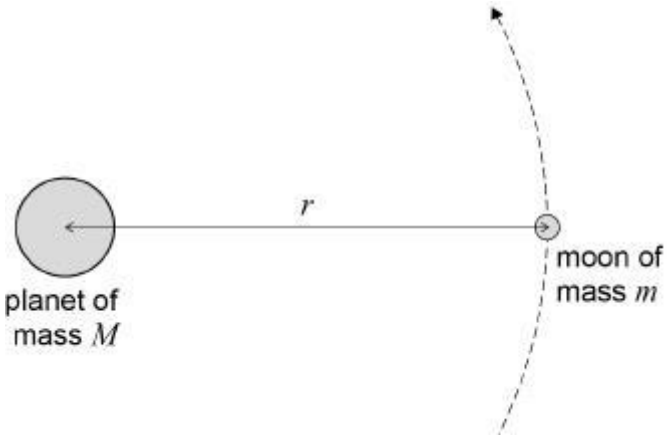
_____

(2)

(Total 10 marks)

Q2.

The figure shows a moon of mass *m* in a circular orbit of radius *r* around a planet of mass M, where $m << M$.



The moon has an orbital period T.
T is related to *r* by

$$T^2 = kr^3$$

where k is a constant for this planet.

(a) Show that $k = \frac{4\pi^2}{GM}$

(3)

Table 1 gives data for two of the moons of the planet Uranus.

Table 1

Name	T / days	r / m
Miranda	1.41	1.29×10^8
Umbriel	4.14	**X**

(b) Calculate the orbital radius **X** of Umbriel.

orbital radius = _____ m

(2)

(c) Calculate the mass of Uranus.

mass = _____ kg

(3)

Table 2 gives data for three more moons of Uranus.

Table 2

Name	Mass / kg	Diameter / m
Ariel	1.27×10^{21}	1.16×10^6
Oberon	3.03×10^{21}	1.52×10^6
Titania	3.49×10^{21}	1.58×10^6

(d) Deduce which moon in **Table 2** has the greatest escape velocity for an object on its surface. Assume the effect of Uranus is negligible.

_____

_____

_____

(3)

(e) A spring mechanism can project an object vertically to a maximum height of 1.0 m from the surface of the Earth.

Determine whether the same mechanism could project the same object vertically to a maximum height greater than 100 m when placed on the surface of Ariel.

_____

_____

_____

(3)

(Total 14 marks)